AST5220
Recombination

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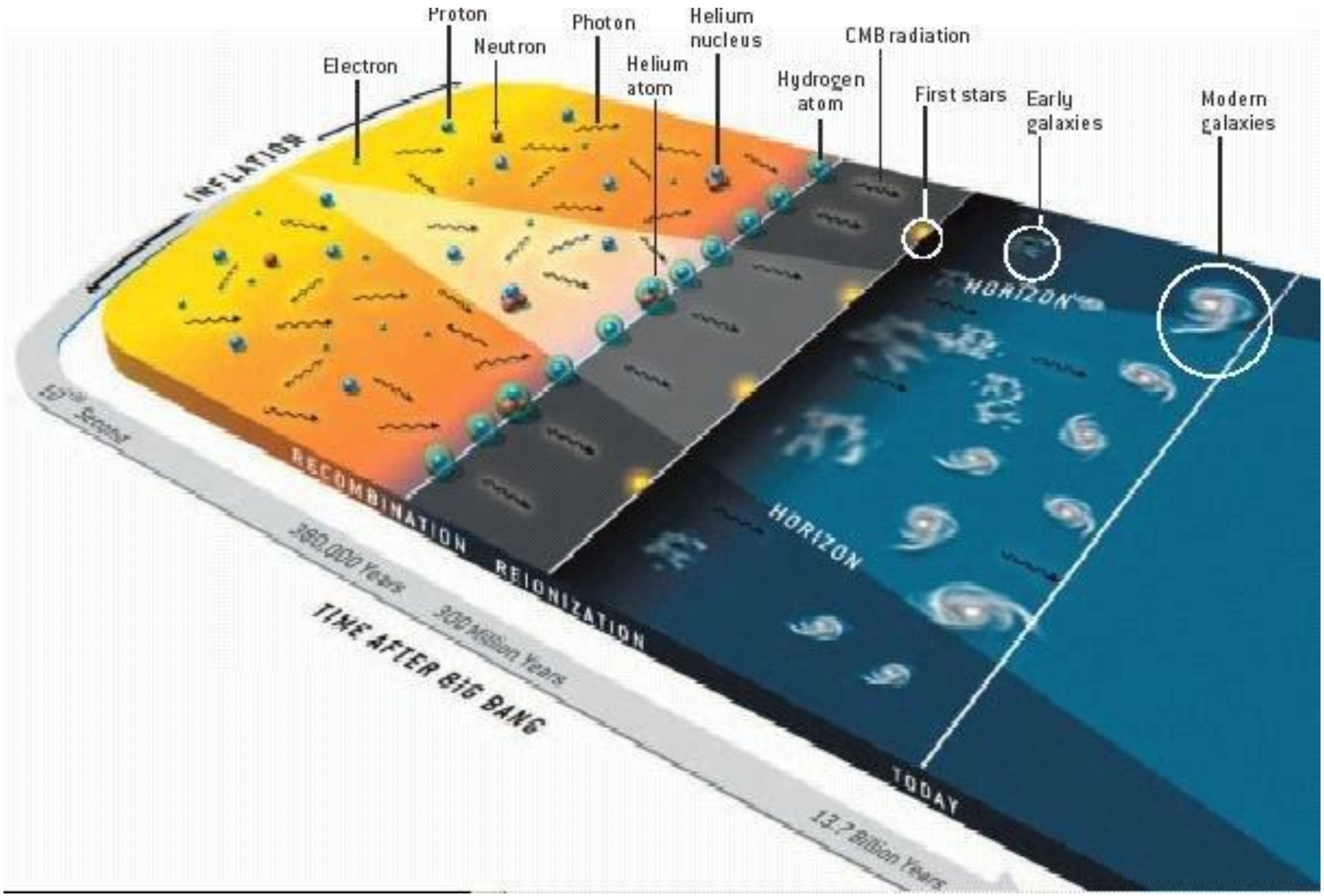
Overview

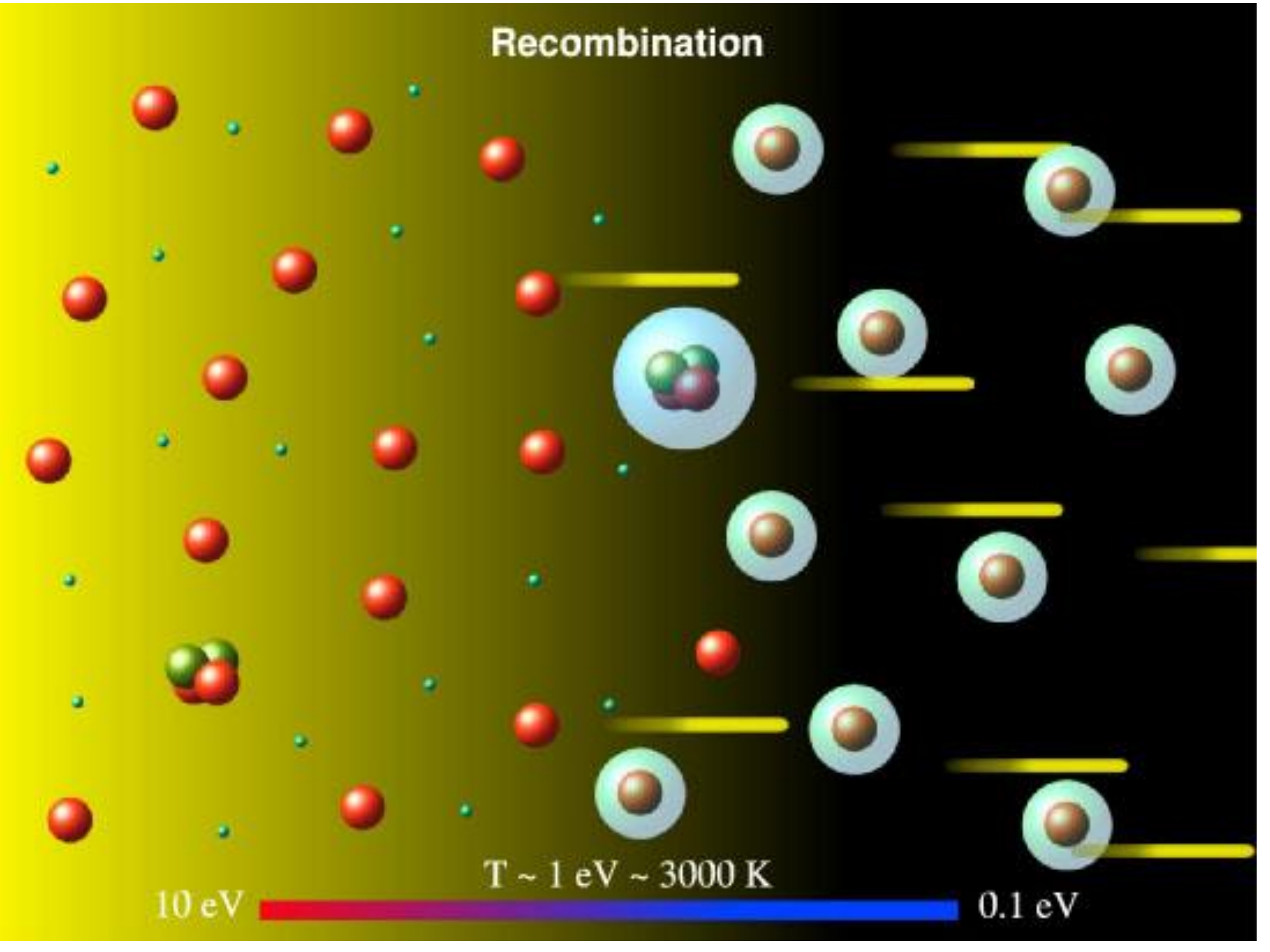
- Some important points:
  - Photons scatter on free electrons
  - In the early universe, the temperature was very hot
    - Too hot for neutral hydrogen to exist; electrons and protons were ripped apart ultraviolet photons
  - Consequently, all electrons in the early universe were free
  - And therefore the universe was opaque
    - It was impossible to see more than ~1 meter ahead, because light was scattered from a different direction
    - Just like walking in a fog
  - But today the universe is transparent
    - Something must have happened at some point

Question: How did the universe become transparent?

Cosmic timeline

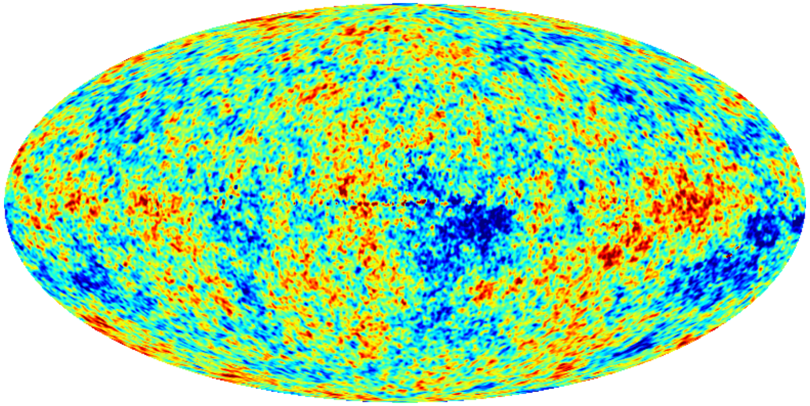


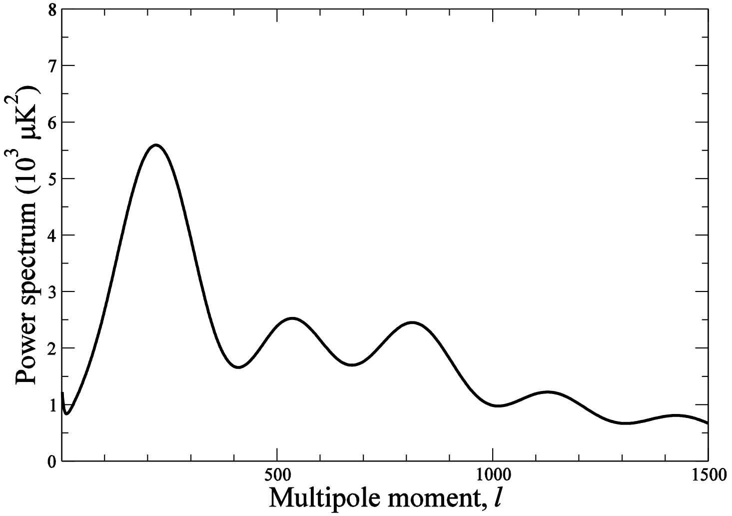
Zoom-in on recomination epoch



Why is this epoch important to us?

- Recall our main goal:
  - We want to predict the CMB power spectrum given cosmological parameters
- The CMB photons we observe today are those that decoupled during recombination
- The fluctuations we see are mainly those that existed at the time of recombination
- We therefore need to know
  - when recombination happened
  - how rapidly it happened





The optical depth

- To describe recombination quantitatively we introduce the concept of optical depth, τ
- Imagine you are looking at a light source through a medium that absorbes light
  - The full intensity of the light source is called I_0
- The intensity you observe at position x is then

$$I(x) = I_0 e^{-\tau(x)}$$

where τ is called the optical depth

- The further away you are, the higher τ is
- The critical position is where $\tau \sim 1$
  - If $\tau >> 1$, you don't see anything
  - If $\tau << 1$, the medium doesn't do anything anyway

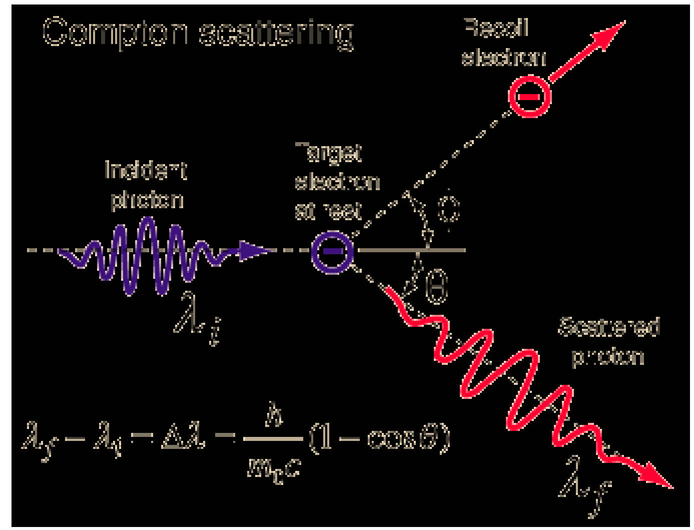
The cosmological optical depth

- In cosmology, the main source of ”absorption” is free electrons through Thompson scattering

- The optical depth of Thompson scattering is given by

$$\tau(\eta) = \int_{\eta}^{\eta_0} n_e \sigma_T a \, d\eta'$$

  where
  - η is conformal time
  - n_e = electron density
  - σ_T = Thompson cross section (”probability of single scattering”?)



- Our main job: Compute $\tau(\eta)$

How do we compute τ?

$$\tau(\eta) = \int_{\eta'}^{\eta_0} n_e \sigma_T a d\eta'$$

- The Thompson cross section is given by particle physics:

$$\sigma_T = \frac{8\pi}{3}\left(\frac{\alpha\hbar}{m_e c}\right)^2 \approx 6.65 \cdot 10^{-25} cm^2$$

- *a* is the scale factor, and a one-to-one function of the conformal time

- The difficult one is the electron density, n_e...

Finding the electron density (1)

- Assume that there are no helium or heavier elements in the universe, only electrons and protons
- That is, there are four constituents in the universe:
  - Protons, p
  - Electrons, e
  - Neutral hydrogen, H
  - Photons, γ
- These interact with each other through a two-particle process

$$p + e \leftrightarrow H + \gamma$$

- We need to find n_e, n_p, n_H and n_γ

Finding the electron density (2)

- And from the last lecture, we know how to do this!

- Recall first the Boltzmann equation for a two-particle process in an expanding universe:

$$a^{-3}\frac{d(na^3)}{dt} = \int \frac{d^3p_1}{(2\pi\hbar)^3 2E_1} \int \frac{d^3p_2}{(2\pi\hbar)^3 2E_2} \int \frac{d^3p_3}{(2\pi\hbar)^3 2E_3} \int \frac{d^3p_4}{(2\pi\hbar)^3 2E_4}$$
$$(2\pi)^4 \delta^{(3)}(p_1 + p_2 - p_3 - p_4)\delta(E_1 + E_2 - E_3 - E_4)|\mathcal{M}|^2$$
$$[f_3 f_4 - f_1 f_2]$$

- After defining

$$n_i = e^{\frac{\mu_i c^2}{kT}} \int \frac{d^3p}{(2\pi)^3} e^{-\frac{E_i}{kT}} \qquad n_i^{(0)} = \int \frac{d^3p}{(2\pi)^3} e^{-\frac{E_i}{kT}}$$

this could (for thermal equlibrium) be written as

$$a^{-3}\frac{d(n_1 a^3)}{dt} = n_1^{(0)} n_2^{(0)} \langle \sigma v \rangle \left\{ \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right\}$$

Finding the electron density (3)

- Recall also that if the reaction rate is much larger than the Hubble expansion rate, then this implies

$$\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} = \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}}$$

- For our proton-electron-hydrogen-photon plasma, this therefore reads

$$\frac{n_p n_e}{n_p^{(0)} n_e^{(0)}} = \frac{n_H n_\gamma}{n_H^{(0)} n_\gamma^{(0)}}$$

- This is the Saha equation on a symbolic form, and now we have to write this out in full. To the blackboard... ☺

The Saha equation

- So, the Saha equation reads

$$\frac{X_e^2}{1 - X_e} \approx \frac{1}{n_b} \left(\frac{m_e T_b}{2\pi\hbar^2} \right)^{3/2} e^{-\frac{\epsilon_0}{k_b T_b}}$$

- This is a good approximation as long as the system is in strong thermodynamic equilibrium

- But when the temperature and density fall, the system eventually goes out of thermodynamic equilibrium

The general equation

- When that happens, one has to go back to the full Boltzmann equation
- If the only involved particles and states were photons, electrons and hydrogen, this would (as usual) look like this:

$$a^{-3}\frac{d(n_e a^3)}{dt} = n_e^{(0)} n_p^{(0)} \langle \sigma v \rangle \left\{ \frac{n_H n_\gamma}{n_H^{(0)} n_\gamma^{(0)}} - \frac{n_e n_p}{n_e^{(0)} n_p^{(0)}} \right\}$$

- However, in real life this quickly becomes messy because of the atomic physics involved:
  - First, e + p → H_{s1} + γ doesn't count
    - the released photon will immediately ionize another nearby hydrogen atom, unless the distance between atoms is very large
  - The most important contribution is e + p → H_{s2} + γ
    - This leads to the Peebles' equation
  - For high precision, helium, litium and excited states must be included
    - This is what Recfast does, and it is *messy*

The Peebles' equation

$$a^{-3}\frac{d(n_e a^3)}{dt} = n_e^{(0)} n_p^{(0)} \langle \sigma v \rangle \left\{ \frac{n_H}{n_H^{(0)}} - \frac{n_e n_p}{n_e^{(0)} n_p^{(0)}} \right\}$$

- Let $n_e = n_b X_e$. Then the left-hand side of the Boltzmann equation becomes

$$a^{-3}\frac{d(n_e a^3)}{dt} = a^{-3}\frac{d(X_e(n_b a^{-3}))}{dt} = n_b \frac{dX_e}{dt}$$

- The right-hand side becomes

$$n_e^{(0)} n_p^{(0)} \langle \sigma v \rangle \left\{ \frac{n_H}{n_H^{(0)}} - \frac{n_e n_p}{n_e^{(0)} n_p^{(0)}} \right\} = n_b \langle \sigma v \rangle \left\{ (1 - X_e) \left(\frac{m_e T}{2\pi\hbar^2} \right)^{3/2} e^{-\frac{\epsilon_0}{kT}} X_e^2 n_b \right\}$$

The Peebles' equation

- Cancelling terms, the equation then reads

$$\frac{dX_e}{dt} = \langle \sigma v \rangle \left\{ (1 - X_e) \left(\frac{m_e T}{2\pi\hbar^2} \right)^{3/2} e^{-\frac{\epsilon_0}{kT}} X_e^2 n_b \right\}$$

- With the correct constants, this leads to the Peebles' equation

- You will solve this numerically in the second milestone of the project
  - The full set of constants are given in the project description

Numerical solution

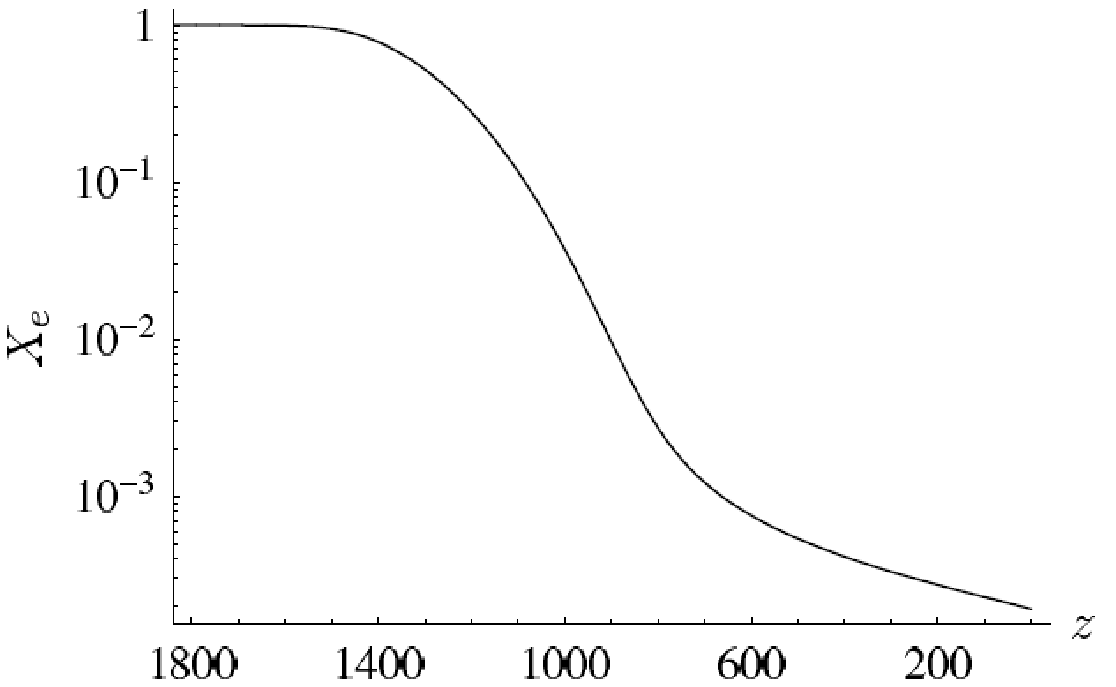


FIG. 1: The free electron fraction X_e as a function of redshift, using the Saha approximation (12) until $z = 1587.4$ where $X_e = 0.99$, and then integrating the Peebles equation (13).

Figure 1 of Callin (2005)

Optical depth and the visibility function

- We now have everything we need to compute the optical depth

$$\tau(\eta) = \int_{\eta'}^{\eta_0} n_e \sigma_T a d\eta'$$

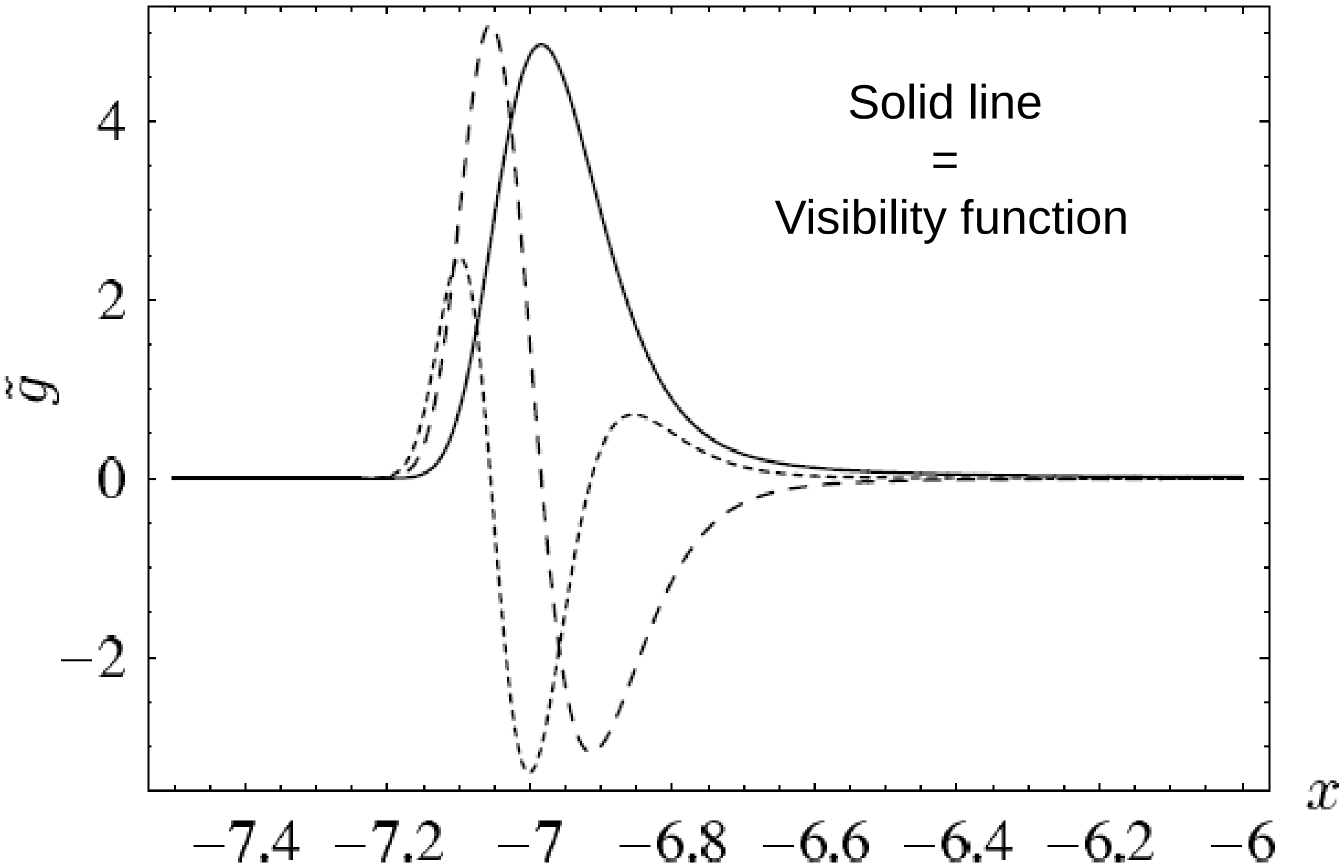
since we finally know $n_e = X_e \, n_b$

- However, we will also need the so-called ”visibility function”

$$g(\eta) = -\frac{d\tau}{d\eta} e^{-\tau(\eta)}$$

- This is simply *the probability for a given observed photon to have scattered at conformal time η*

Optical depth and the visibility function



z ~ 1100

Summary

- We need to know the recombination history of the universe in order to predict the CMB spectrum
  - Specifically, we need the optical depth, τ, and the visibility function, g
- To find these, we must solve the Boltzmann equation for the electron number density in the early universe
  - At early times, use the Saha approximation
  - At late times, use the Peebles' equation
- In order to do this job properly, one should really take into account heavier elements (helium, litium etc.) and many excitation states of each atom
  - Not trivial, and fortunately somebody else has spent a lot of time on this, and written Recfast
  - We will be happy with the simpler approximations above in this course
- Recombination (and reionization) is messy... ☺